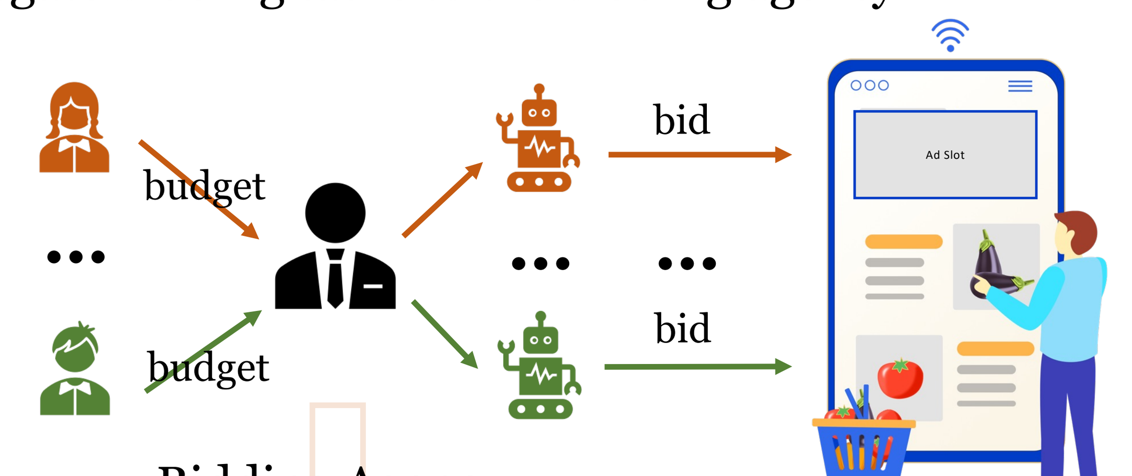
Coordinated Dynamic Bidding in Repeated Second-Price Auctions with Budgets

Yurong Chen[*,1], Qian Wang[*,1], Zhijian Duan[1], Haoran Sun[1], Zhaohua Chen[1], Xiang Yan[2], Xiaotie Deng[1]

[*]Equal Contribution. [1]Peking University [2]Huawei TCS Lab

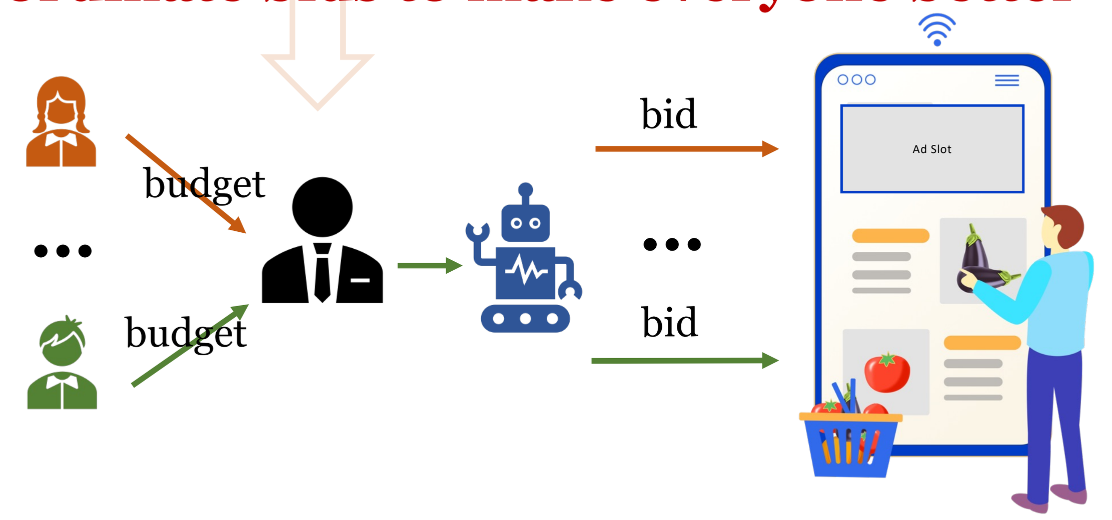
Introduction

Online repeated **second-price** auctions. Advertisers delegate bidding tasks to the bidding agency



Bidding Agency:
information of all her clients

Coordinate bids to make everyone better



Main Results: Propose Online Coordinated Bidding Algorithms

1. **Theoretically** and **Experimentally** guarantee every bidder gains **better** than they independently bid
   - Under assumptions of strong monotonicity.
2. **Game-Theoretic Property Analysis** in symmetric cases:
   - Coalition welfare maximization;
   - incentive to misreport budgets.

Settings

At round $t = 1, \dots, T$:

- Bidder k in coalition $\mathcal{K}$: budget $B_k = \rho_k T$, value $v_{k,t}$;
- Highest bid outside coalition: d_t^O.

$v_{k,t}$s and d_t^O: each follows stationary distributions, i.i.d. across rounds.

Feasible: not exceed the budgets in all cases.

Difficulties

- Interplay of bidders in dynamic repeated settings:
  - multi-player online games: a bidder's utility is influenced by others' bids.
- Multi-benchmark comparison:
  - Everyone is better off using coordinated bidding.

Allow monetary transfer among bidders?
So that maximizing coalition welfare is sufficient.

Not Reasonable.

Bidders participate in auctions to win more impressions for their ads, rather than simply conducting financial investments.

Benchmark

Individual Adaptive Pacing (IP): When everyone inside the coalition uses adaptive pacing [Balseiro et al'19] independently:

At round t: bidder k bids $b_{k,t} = v_{k,t}/\lambda_{k,t}$

Update: $\lambda_{k,t+1} = P_{[0,\bar{\lambda}]}\left(\lambda_{k,t} - \epsilon(\rho_k - z_{k,t})\right)$

- Optimal individual bidding strategies in stationary and adversary environments.
- Expected expenditure per round is strongly monotone: converging to some **equilibria**.

Our Benchmark!

Technique 1 – Algorithm Design

1. A form of optimal strategies: select one representative to bid in each round.

- A fair rule to select representatives in each round — Important!
- Budget management strategies — Pacing

Coordinated Pacing (CP): Guarantee, Coalition Maximization, Budget-IC in Symmetric Cases

Natural extension of adaptive pacing

$$k^* \in \arg\max_{k \in \mathcal{K}} \min\left\{\frac{v_{k,t}}{1+\lambda_{k,t}}, \bar{B}_{k,t}\right\}$$

Bidder k:

Bid $b_{k,t} = \min\left\{\frac{\mathbb{1}\{k=k^*\}v_{k,t}}{1+\lambda_{k,t}}, \bar{B}_{k,t}\right\}$

Update:

Pacing Parameter: $\lambda_{k,t+1} = P_{[0,\bar{\lambda}]}\left(\lambda_{k,t} - \epsilon(\rho_k - z_{k,t})\right)$

Remaining Budgets: $\bar{B}_{k,t+1} = \bar{B}_{k,t} - z_{k,t}$

- Easy to analyze the game-theoretic properties
- Fails to guarantee in General Cases!
- Selection rule is IMPORTANT!

Hybrid Pacing (HP): Guarantee in General. Coalition Maximization in Symmetric Cases

Inner Auction to select the representative

Pseudo parameter $\lambda_{k,t}$ for bidding inside the coalition

Bidder k:

$b_{k,t}^I = \min\left\{\frac{v_{k,t}}{1+\lambda_{k,t}}, \bar{B}_{k,t}\right\}$ $\quad k^* \in \arg\max_{k \in \mathcal{K}} b_{k,t}^I$

Pacing parameter $\mu_{k,t}$ for bidding in the real auction

Bid $b_{k,t}^O = \min\left\{\frac{\mathbb{1}\{k=k^*\}v_{k,t}}{1+\mu_{k,t}}, \bar{B}_{k,t}\right\}$

Update:

Pseudo Parameter: $\lambda_{k,t+1} = P_{[0,\bar{\lambda}]}\left(\lambda_{k,t} - \epsilon(\rho_k - z'_{k,t})\right)$

Pseudo Remaining Budget:

$$z'_{k,t} = \mathbb{1}\{b_{k,t}^I \geq z_{k,t}\}\max(z_{k,t}, x_{k,t}d_{k,t}^I)$$

Pacing Parameter: $\mu_{k,t+1} = P_{[0,\lambda_{k,t+1}]}\left(\mu_{k,t} - \epsilon(\rho_k - z_{k,t})\right)$

Remaining Budgets: $\bar{B}_{k,t+1} = \bar{B}_{k,t} - z_{k,t}$

- The inner selection simulates the same opportunities as in IP, so as to provide fair selection rule.
- Guarantee profit gains for per-member in **General Cases**!

References:

Balseiro, S. R. and Gur, Y. Learning in repeated auctions with budgets: Regret minimization and equilibrium. Management Science, 2019

Technique 2&3 – Algorithm Analysis

2. Strong monotonicity makes sure the bidders' utilities converges:

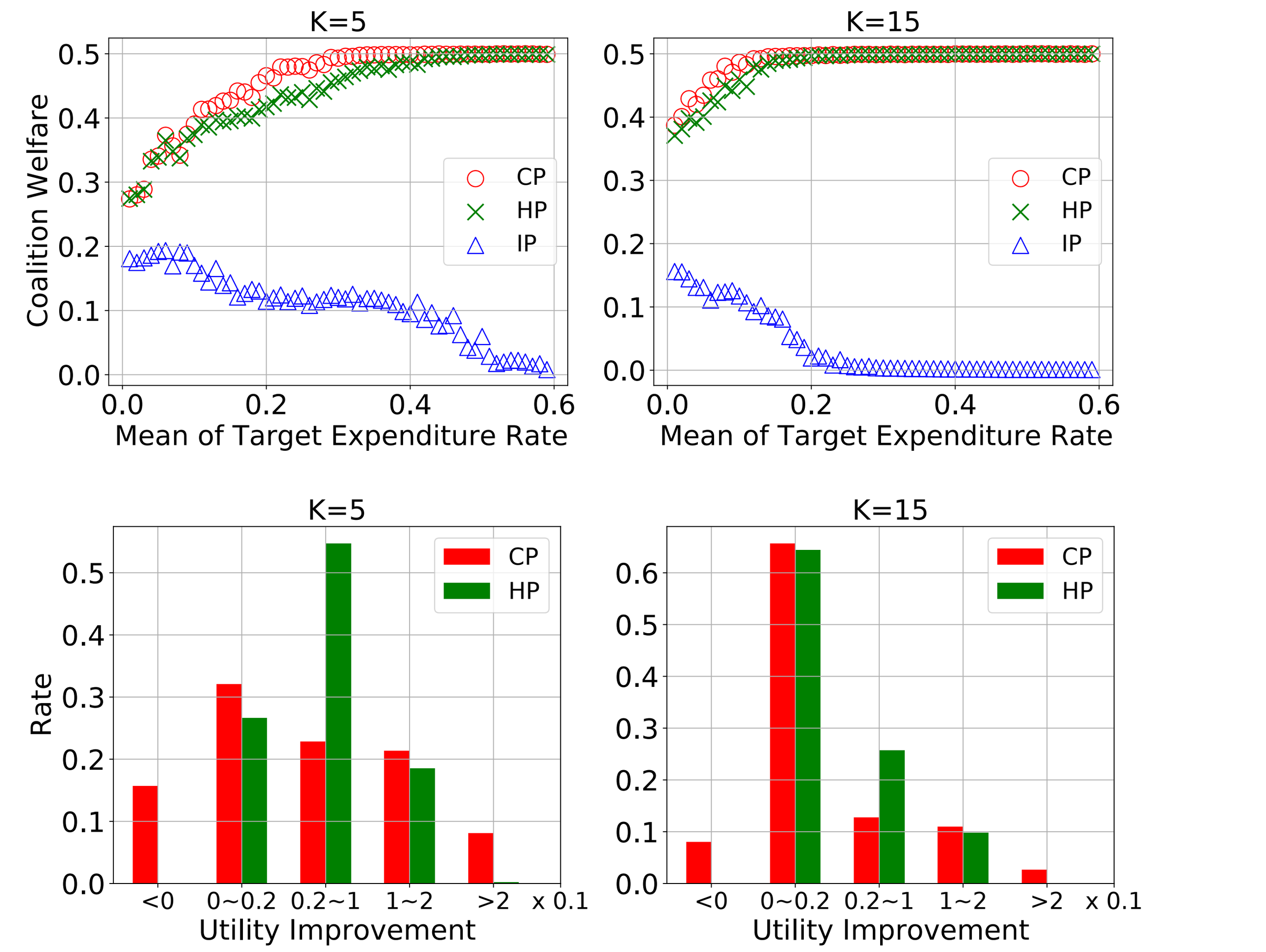
Comparison to benchmark

⇩

Comparison of per-person equilibria utilities

3. Per-person equilibrium utility comparison:
   Enable game-theoretic property analysis in symmetric cases.

Experiments on Real Data



Dataset: https://contest.ipinyou.com/

- Values: normalized bids in dataset to be in [0,1] .
- Highest bid outside coalition: $d_t^O \sim \mathcal{N}(0.5, 0.2)$.
- Each point: $\bar{\rho} = 0.01\alpha, \alpha \in [100]$. Sample each bidder's ρ_k from $\mathcal{N}(\bar{\rho}, 0.1)$.

For more details, scan to see the full paper ☺

contact at: chenyurong@pku.edu.cn; charlie@pku.edu.cn; xiaotie@pku.edu.cn